# Distributed Multipole Analysis of Gaussian wavefunctions <br> GDMA version 2.2.02 

Anthony J. Stone

## 1 Introduction

It is now possible to carry out Distributed Multipole Analysis of wavefunctions calculated using either the Gaussian system of programs[1], using the formatted checkpoint file produced by Gaussian, or the CamCASP package[2]. If you find any bugs, please send email to Anthony Stone, ajs1@cam.ac.uk. Comments and corrections would also be welcome. Instructions given here refer to the use of the program with Gaussian. Most of the information also applies to the CamCASP package; for the differences, see the CamCASP documentation.

The recommended procedure for using the program is to construct a small data file of the following form:

FILE checkpointfile [DENSITY density-type]
[NAMES
List of atom names, on one or more lines]
MULTIPOLES
subcommands, described below
START
The keywords shown in uppercase may be typed in upper, lower or mixed case. The DENSITY specification is optional; the default is to read the SCF density matrix from the checkpoint file. (Here and later, square brackets denote optional items. The square brackets themselves are not part of the data.) Any other density matrix that appears in the checkpoint file may be specified; if the name contains spaces it must be enclosed in single or double quotes. The NAMES command is optional, but may be needed for some DMA options. If it is provided, the atom names should be listed, separated by spaces, in the order that the atoms appear in the checkpoint file. If the NAMES command is omitted, each atom is given a name which is the chemical symbol corresponding to its atomic number. The MULTIPOLES command may be repeated to obtain DMAs with different options, for example with different choices of multipole sites. The FILE command may be repeated to read another checkpoint file, or a different density from the same file.

Other options that may appear are:
VERBOSE
This may precede the FILE command, and causes the program to print information about the data read from the checkpoint file. It is primarily for debugging purposes and should not normally be needed.

ANGSTROM
This should follow the FILE command unless the atom coordinates in the formatted checkpoint file are in Angstrom instead of atomic units. Any coordinates given in subsequent MULTIPOLES
sections of the data (see below) will then be taken to be in Angstrom, and coordinates printed by the program will be given in Angstrom also, instead of atomic units.

BOHR
This can be used to cancel the effect of a previous ANGSTROM command, but will not normally be needed.

AU
Multipole moments printed by the program will be in atomic units, $e a_{0}^{n}$ for moments of rank $n$. (See sec. 2.1 below for the definition of rank.) This is the default.

SI
Multipole moments printed by the program will be in SI units, $\mathrm{Cm}^{n}$ for moments of rank $n$, but multiplied by a factor $10^{20+10 n}$. For example, a dipole moment (rank 1) with a value of $0.7 \times 10^{-30} \mathrm{Cm}$ will be printed as 0.7 .

## 2 Distributed Multipole Analysis

Distributed Multipole Analysis or DMA is a technique for describing a molecular charge distribution by using local multipoles at a number of sites within a molecule. It gives a much more accurate representation of the charge density than a single-point multipole expansion, even for small molecules. For details see $\S 2.1$ below. The sites are usually the nuclei, but existing sites can be omitted and new sites added. The multipoles are the charges, dipoles, quadrupoles, etc., up to some limit, which can be as high as rank 10 . The limit need not be the same on every atom. For example the following calculation would give multipoles up to rank 4 (hexadecapole) on the oxygen atom, but only up to rank 1 on the hydrogen atoms. (Higher moments on hydrogen atoms are usually very small.) In a case like this, the higher moments on the H atoms are transferred to the nearest atom with a higher limit, here the oxygen, and the overall molecular moments are correct up to the highest rank specified, rank 4 in this case. (See below, p. 8, for an explanation of the RADIUS command.)

MULTIPOLES
LIMIT 4
LIMIT 1 H
RADIUS H 0.35
START
When used with the output file from a B3LYP calculation on water, using the aug-cc-pVTZ basis set, the output is as shown on the next page. On the oxygen one gets the charge ( Q 00 ), the components of the dipole $(\mathrm{Q} 1)$, the quadrupole $(\mathrm{Q} 2)$, octopole $(\mathrm{Q} 3)$ and hexadecapole $(\mathrm{Q} 4)$ moments, while on the hydrogen one gets only the charge and dipole moment. The components are expressed in terms of spherical harmonic definitions (see below). Only the non-zero moments are printed.

```
0 x = 0.000000 y = 0.000000 z = 0.116982
        Maximum rank = 4 Relative radius = 0.650
    QOQ = -0.276214
|Q1| = 0.484939 Q10 = -0.484939
|Q2| = 1.669447 Q20 = 0.126597 Q22c = -1.664640
|Q3| = 3.360888 Q30 = 1.303370 Q32c = 3.097869
|Q4| = 6.929535 Q40 = -3.612674 Q42C = -5.030396 Q44C = 3.108402
H x = 0.000000 y = 0.763556 z = -0.467927
        Maximum rank = 1 Relative radius = 0.350
    Q00 = 0.138107
|Q1| = 0.031927 Q10 = 0.031774 Q11s = -0.003124
H
H x = -0.000000 y = -0.763556 z = -0.467927
        Maximum rank = 1 Relative radius = 0.350
    QOQ = 0.138107
|Q1| = 0.031927 Q10 = 0.031774 Q11s = 0.003124
Total multipoles
referred to origin at x = 0.000000, y = 0.000000, z = 0.000000
    Q00 = -0.000000
|Q1| = 0.726696 Q10 = -0.726696
|Q2| = 2.164751 Q20 = -0.276239 Q22c = -2.147054
|Q3| = 3.481250 Q30 = 1.811733 Q32c = 2.972663
|Q4| = 5.620281 Q40 = -2.635421 Q42c = -3.021910 Q44C = 3.938297
```


### 2.1 Definitions of multipole moments

The spherical harmonic multipole $Q_{l k}$ is defined as

$$
Q_{l k}=\int R_{l k}(\mathbf{r}) \rho(\mathbf{r}) \mathrm{d}^{3} \mathbf{r}
$$

where $\rho(\mathbf{r})$ is the total charge density, and the regular solid harmonics $R_{l k}(\mathbf{r})$ are given in Table 1. For those who prefer the cartesian definitions of multipole moments the relationships between spherical harmonic and cartesian definitions up to hexadecapole are given in Table 2. There are fewer components of the multipole moments in the spherical form than in the cartesian form (except for the charge and dipole) as the components of the cartesian multipoles are not all independent.

The quantities listed as $|\mathrm{Qn}|$ in the program output are the magnitudes of the multipole moments, defined by

$$
\left|Q_{n}\right|=\left(\sum_{k}\left|Q_{n k}\right|^{2}\right)^{1 / 2}
$$

They are independent of axis system, and $\left|Q_{1}\right|$ coincides with the usual expression for the magnitude of the dipole moment.

As well as the distributed multipoles, the program prints the total multipoles referred to the

Table 1: Regular solid harmonics

$$
\begin{aligned}
& R_{00}=1 \quad R_{30}=\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) \quad R_{40}=\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
& R_{10}=z \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& R_{11 c}=x \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& R_{20}=\frac{1}{2}\left(3 z^{2}-r^{2}\right) \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& R_{21 c}=\sqrt{3} x z \\
& \begin{array}{rlrl}
R_{30} & =\frac{1}{2}\left(5 z^{3}-3 z r^{2}\right) & R_{40} & =\frac{1}{8}\left(35 z^{4}-30 z^{2} r^{2}+3 r^{4}\right) \\
R_{31 c} & =\sqrt{\frac{3}{8}} x\left(5 z^{2}-r^{2}\right) & R_{41 c} & =\sqrt{\frac{5}{8}}\left(7 x z^{3}-3 x z r^{2}\right) \\
R_{31 s} & =\sqrt{\frac{3}{8}} y\left(5 z^{2}-r^{2}\right) & R_{41 s}=\sqrt{\frac{5}{8}}\left(7 y z^{3}-3 y z r^{2}\right) \\
R_{32 c}=\sqrt{\frac{15}{4}} z\left(x^{2}-y^{2}\right) & R_{42 c}=\sqrt{\frac{5}{16}}\left(x^{2}-y^{2}\right)\left(7 z^{2}-r^{2}\right) \\
R_{32 s}=\sqrt{15} x y z & R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
R_{33 c}=\sqrt{\frac{5}{8}}\left(x^{3}-3 x y^{2}\right) & R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
R_{33 s}=\sqrt{\frac{5}{8}}\left(3 x^{2} y-y^{3}\right) & R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{array} \\
& R_{21 s}=\sqrt{3} y z \\
& R_{22 c}=\sqrt{\frac{3}{4}}\left(x^{2}-y^{2}\right) \\
& R_{22 s}=\sqrt{3} x y \\
& R_{42 s}=\sqrt{\frac{5}{4}} x y\left(7 z^{2}-r^{2}\right) \\
& R_{43 c}=\sqrt{\frac{35}{8}} z\left(x^{3}-3 x y^{2}\right) \\
& R_{43 s}=\sqrt{\frac{35}{8}} z\left(3 x^{2} y-y^{3}\right) \\
& R_{44 c}=\sqrt{\frac{35}{64}}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right) \\
& R_{44 s}=\sqrt{\frac{35}{4}}\left(x^{3} y-x y^{3}\right)
\end{aligned}
$$

origin of the coordinate frame. Components which are zero are not printed, except for the total charge.
All moments are usually given in atomic units, $e a_{0}^{k}$ for moments of rank $k$, but it is possible to have them printed in SI units instead (SI command, p. 2).

### 2.2 Multipole allocation algorithms

The original distributed multipole analysis, as set out in the bibliography[3-5], starts from the expansion of the electron density in terms of the primitive gaussian basis functions $\chi_{i}(\boldsymbol{r})$ :

$$
\begin{equation*}
\chi_{i}(\boldsymbol{r})=N_{i} x_{i}^{a_{i}} y_{i}^{b_{i}} z_{i}^{c_{i}} \exp \left[-\zeta_{i}\left(\boldsymbol{r}_{i}\right)^{2}\right] \tag{1}
\end{equation*}
$$

where $\boldsymbol{r}_{i}=\boldsymbol{r}-\boldsymbol{A}_{i}$ is the electron position relative to the position $\boldsymbol{A}_{i}$ of the primitive Gaussian, $\zeta_{i}$ is the exponent and $N_{i}$ is a normalising factor. The electron density is then

$$
\begin{equation*}
\rho(\boldsymbol{r})=\sum_{i j} D_{i j} \chi_{i}(\boldsymbol{r}) \chi_{j}(\boldsymbol{r}) . \tag{2}
\end{equation*}
$$

Boys[6] showed that a product of gaussians $\chi_{i}(\boldsymbol{r}) \chi_{j}(\boldsymbol{r})$ can be expressed as a gaussian function centred at $\boldsymbol{P}_{i j}=\left(\zeta_{i} \boldsymbol{A}_{i}+\zeta_{j} \boldsymbol{A}_{j}\right) /\left(\zeta_{i}+\zeta_{j}\right)$. The multipole moments of the associated charge distribution can be calculated exactly, and in standard DMA they are represented by a multipole expansion about the nearest site. This procedure is very fast and efficient, and gives an excellent representation of the molecular charge distribution.
If the basis set includes very diffuse functions, however, overlap densities involving them extend over many atoms. In carbon dioxide, for instance, the product of a diffuse $s$ function on

Table 2: Relationship between spherical-tensor and cartesian-tensor form of multipole moments

$$
\begin{aligned}
\mu_{z} & =Q_{10} \\
\mu_{x} & =Q_{11 c} \\
\mu_{y} & =Q_{11 s} \\
\Theta_{x x} & =-\frac{1}{2} Q_{20}+\frac{1}{2} \sqrt{3} Q_{22 c} \\
\Theta_{y y} & =-\frac{1}{2} Q_{20}-\frac{1}{2} \sqrt{3} Q_{22 c} \\
\Theta_{z z} & =Q_{20} \\
\Theta_{x y} & =\frac{1}{2} \sqrt{3} Q_{22 s} \\
\Theta_{x z} & =\frac{1}{2} \sqrt{3} Q_{21 c} \\
\Theta_{y z} & =\frac{1}{2} \sqrt{3} Q_{21 s}
\end{aligned}
$$

one O atom and a diffuse $p_{z}$ function with slightly larger exponent on the C atom would be formally centred between the C and O , but slightly nearer to the C atom, and would give rise to charge and dipole contributions which would be allocated to the C atom. However, much of the electron density would in fact be in the region of the O atom, and it would extend to some extent over all three atoms. It would be more satisfactory to apportion the charge density between the atoms in some way.

Methods for doing this have been developed in recent years for handling the integrals of density functional theory. Version 2 of the GDMA program[7] uses these methods to calculate the multipole contributions arising from the overlap of diffuse primitive functions. Such an approach is not needed for compact basis functions (those with large $\zeta$ ) because the overlap densities involving such functions are well localised in space, and their allocation to the nearest multipole site is entirely satisfactory. Moreover the grid-based quadrature used for the diffuse functions is less satisfactory for highly-peaked functions.

Consequently the program now uses both methods. If the sum of exponents $\zeta_{i}+\zeta_{j}$ for a pair of primitive functions $\chi_{i}$ and $\chi_{j}$ is less than a switch value $Z$, the grid-based analysis is used, and otherwise the original DMA method is used. The value of $Z$ can be specified, as can the parameters for the grid-based quadrature, but the default values should be satisfactory in most cases. The original DMA can be obtained by setting $Z=0$.

### 2.3 Options for multipole analysis

The MULTIPOLES command has various subcommands, two of which, LIMIT and RADIUS, were included in the example dataset. The LIMIT command sets the maximum rank for the DMA analysis. For example, LIMIT 4 instructs the program to calculate multipoles up to hexadecapole only. The highest rank which can be calculated for non-linear molecules is 10 , i.e. LIMIT 10. LIMIT 1 would calculate only charge and dipole on each site, LIMIT 2 would include the quadrupole moments, and so on. It is also possible to set different limits for different sites. For example

MULTIPOLES
LIMIT 4
LIMIT 1 H
START
would calculate multipoles up to hexadecapole on all sites except those called ' H ', where the expansion would terminate with the dipole moments. If this dataset is used for water, as in the example, the effects of the higher moments on the H atoms are transferred to the O atom, so that the overall multipole moments for the whole molecule are correct up to the highest rank specified. This is usually a sensible procedure for H atoms, where the higher moments are small. The limit for named atoms is applied to all atoms with the name given, so if different limits are wanted for atoms of the same element, the NAMES command must be used to give them distinctive names. The general form of the LIMIT command is 'LIMIT $n$ name name ...'.

The list of sites can be modified using ADD and DELETE. Extra sites can be included in the analysis by using the ADD command:

ADD name $x$ yz [LIMIT $n$ ] [RADIUS $r$ ]
where $x, y$ and $z$ are the cartesian coordinates of the new site (in atomic units, unless the ANGSTROM command has been used). Items in square brackets are optional. The optional LIMIT $n$ sets the maximum rank for this site; if it is omitted, the current maximum rank is used. The RADIUS option, if present, sets the radius (see below). If it is omitted, a value of $0.65 \AA$ is used.

DELETE removes all sites with the name specified. For example,
MULTIPOLES
DELETE H
START
would remove from the analysis all sites called ' H '. This would sometimes be done in large molecules to avoid an over-proliferation of expansion centres. For example one might want to avoid using the methyl hydrogen atoms as multipole sites in a large molecule. To delete the methyl hydrogens in methanol but retain the OH hydrogen (a common situation where hydrogen bonding is involved) it would be necessary to use different names for the methyl and OH hydrogens, perhaps ' H ' and 'Ho' respectively, and then DELETE H would have the required effect.

If you use DELETE ALL then all sites are removed. In this case you will need to use ADD to provide at least one site for the expansion. This provides a way to get a single-site expansion referred to a site other than the origin of coordinates. However a simpler way is to use the command

ORIGIN $o_{x} o_{y} o_{z}$
to specify the origin for the overall multipole moments.
It should perhaps be emphasized once again that whatever sites are added or deleted, the overall multipole moments will be correct up to the maximum rank of any site that remains. However, if too few sites are used, the resulting multipole expansion of the electrostatic potential or of the electrostatic interaction between molecules may be inaccurate. See my book for a fuller discussion[5].
The switch between the original DMA algorithm and the new grid-based quadrature can be controlled by the command

SWITCH Z
The default value is 4 .
The GRID command controls parameters for the quadrature grid:
GRID options
The options are
LEBEDEV $n$
Use Lebedev angular quadrature[8] with at least $n$ points for each atom. Lebedev quadrature is the default, but this command is needed if the number of points is to be changed from the default of 590 . Note that a high value is needed, especially if high-rank multipoles are required, because they are quite sensitive to the angular behaviour of the charge density.

GAUSS-LEGENDRE $n$
Use Gauss-Legendre quadrature in $\theta$ and equally-spaced intervals in $\phi$, with the total number of points chosen to be at least $n$.

RADIAL $n$

Use $n$ points for the radial quadrature (default 80). This value needs to be large, especially if high-rank multipoles are required, because they are strongly influenced by the electron density at large distances from the nuclei.

SMOOTHING $m$
This specifies the integer parameter used in the Becke procedure[9] for calculating the weights of points assigned to each atom. The default value is 3 ; this is smaller than is usually recommended for DFT calculations, because a softer transition between atoms is more suitable for multipole moment calculations.
It is usual in setting up the grid for grid-based quadrature to assign different radii to different elements. These radii are used in scaling the radial quadrature grid[10]. The radii used in standard density-functional calculations are usually Bragg-Slater covalent radii[11], though the value used for hydrogen is twice the Bragg-Slater value. These are used here in the usual way for scaling the radial grid, but they are not suitable for partitioning the density between atoms in the distributed multipole analysis - they lead to large and implausible atom charges - and for this purpose the GDMA program sets all the atom radii to be equal by default, except for hydrogen (see below). The use of equal radii for all sites is the most efficient choice from the point of view of convergence of the resulting multipole expansion of the electrostatic potential[5]. The radius used is $0.65 \AA$. Additional sites specified using the ADD command are also assigned a radius of $0.65 \AA$ unless some other value is explicitly specified. Different values may be assigned to existing sites using the option

RADIUS $n_{1} r_{1} n_{2} r_{2} \ldots$
which assigns the radius $r_{1}$ to every site with the name $n_{1}$, radius $r_{2}$ to sites called $n_{2}$, and so on. The values chosen for the radii are not critical - any reasonable choice will give multipoles that describe the electrostatic potential accurately - but some experimentation may be needed to obtain atom charges that correspond to chemical intuition. In particular, a radius of $0.325 \AA$ for hydrogen has been found to give more acceptable values than the default of $0.65 \AA$, especially where hydrogen-bonding is involved, and this is now the default for hydrogen. That is, any atom site with nuclear charge 1 is initially assigned a radius of $0.325 \AA$. Some experimentation may be needed for other atoms; for example, a radius of $1.11 \AA$ has been found suitable for Cl in methyl chloride.
The command
PUNCH [APPEND] [RANK p] [punch-file]
causes the program to produce a summary of output in the specified file in a form suitable for reading in to the Orient program[12]. This output includes only multipoles up to rank $p$ (default 5), or less for any site for which a lower limit has been specified. The numerical values will be in atomic units, even if printed values are in SI. If the file name is omitted, and a punch file has been specified for an immediately preceding MULTIPOLES command in the same job, the punch output is appended to the same file.

APPEND is optional. If it is omitted, any existing file with the specified name is overwritten, unless it has already been opened for punch output from an immediately preceding MULTIPOLES command in the same job, in which case APPEND is assumed. If APPEND is specified or assumed, and the file exists, the new output is appended to it.

The MULTIPOLES command may be used any number of times to construct different multipole
representations of the electron density. If the checkpoint file contains more than one density matrix - perhaps an SCF density and an MP2 density - the FILE command can be repeated, specifying a different density, to re-read the checkpoint file. Subsequent MULTIPOLES commands will then use that density.

## 3 Important notes

### 3.1 Algorithms

The algorithm originally used by the program is both exact and very fast, because it uses an exact and very efficient Gauss-Hermite quadrature. The new version normally uses a gridbased quadrature for integrations involving diffuse functions, and is very much slower, because it is necessary to use a fine grid, and even with a fine grid it is not exact. An estimate of the errors may be obtained by carrying out a calculation for a symmetrical molecule where some multipoles should be zero by symmetry. They will normally be much smaller than any uncertainties arising from approximations in the original ab initio calculation.

The original integration algorithm can be used by setting "SWITCH 0" as explained above. The calculation then runs with full speed and accuracy. The multipoles may not correspond as well with chemical intuition, but the electrostatic potential will still be accurate, except possibly in the region of hydrogen-bonded H atoms. The difference in electrostatic potential between the two methods may be explored using my Orient program.

### 3.2 Point-charge models

If you need a point-charge model, do not use the point charges from GDMA. The effects of atomic distortions caused by chemical bonding are described in GDMA by the dipoles and higher moments, and simply omitting those will give very poor results. For good results you need to include dipoles and quadrupoles at least. If you can only handle point charges, the recommended procedure is to use the Mulfit program of Winn et al. [13, 14], which approximates the atomic dipoles and higher moments by means of charges on neighbouring atoms. It can also be used to refine a more detailed description, for example with multipoles up to quadrupole, by approximating the contributions of the octopoles and hexadecapoles. Figure 1 illustrates the improvement that can be obtained in this way. The figure shows maps of the difference between various model potentials and the potential due to an accurate DMA, including multipoles up to hexadecapole. These maps were prepared using the Orient program[12]. The potential ranges from about -1 V near the O atom to about +1 V near the $\mathrm{NH}_{2}$ hydrogen atoms. The potentialderived charges and the charges fitted to the rank-4 DMA lead to errors of about $5 \mathrm{~kJ} \mathrm{~mol}^{-1}$ for a charge of $0.5 e$, while the plain DMA charges give a significantly worse result. The models using multipoles up to quadrupole both have errors of the order of $1 \mathrm{~kJ} \mathrm{~mol}^{-1}$; here fitting to incorporate the effects of octopole and hexadecapole moments improves the description only slightly.

The Mulfit package is included with the GDMA distribution, by permission of its authors. See the instructions included with that package for details of its use.


Figure 1: Point-charge and distributed-multipole models for the electrostatic potential of formamide on a surface at 1.8 van der Waals radii. (a) Potential-derived charges, (b) DMA charges, (c) charges fitted to DMA rank 4 using Mulfit, (d) DMA up to quadrupoles, (e) multipoles up to quadrupole fitted to DMA rank 4 using Mulfit. For a charge of $0.5 e, 0.1 \mathrm{~V}$ corresponds to about $5 \mathrm{~kJ} \mathrm{~mol}^{-1}$.

## 4 Example data files

A number of examples are provided in the examples directory and its subdirectories, together with output files. The script run.tests in the examples directory runs these calculations automatically and compares the results with the output files provided.

## 5 Program limits

In version 2.2.02, the arrays used by the program are allocated as required, so there should be no problems with large molecules or large basis sets. The maximum number of sites is arbitrarily set at 16 more than the number of atoms, and this should be sufficient, but if more sites are needed it will be necessary to increase the value of nextra near the top of the gdma.f90 file and recompile.

## 6 Citation

Please use the citation as in Ref. 7 when referring to the program.

## 7 Revision notes

Version 2.2.02 uses dynamic allocation of arrays, so that there are no arbitrary limits on the size of the molecule or the basis set. There is an arbitrary limit on the number of sites, as noted above, but it should be adequate for most if not all cases.

Version 2.2 incorporates the minor but significant change that the radius assigned by default to hydrogen atoms is half the default radius for other atoms. Calculations involving H atoms with default radii will give results that are different from earlier versions.

Version 2.1 provides for printing multipole moments in SI units as an alternative to the standard atomic units.

Version 2.0 This version of the program can handle basis functions up to $g$. It also provides for real-space apportionment of the charge density arising from low-exponent primitives, instead of the allocation of all multipoles from a particular primitive-function overlap density to the nearest site.

Version 1.3 Explicit type declarations introduced. This dealt with an apparent bug in the Portland compiler, which gave wrong results when implicit declarations were used.

Version 1.2 Modified to handle new features in the Gaussian03 formatted fchk files.

## References

[1] M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R. Cheeseman, J. A. Montgomery, Jr., T. Vreven, K. N. Kudin, J. C. Burant, J. M. Millam, S. S. Iyengar, J. Tomasi, V. Barone, B. Mennucci, M. Cossi, G. Scalmani, N. Rega, G. A. Petersson, H. Nakatsuji, M. Hada, M. Ehara, K. Toyota, R. Fukuda, J. Hasegawa, M. Ishida, T. Nakajima, Y. Honda, O. Kitao, H. Nakai, M. Klene, X. Li, J. E. Knox, H. P. Hratchian, J. B. Cross, V. Bakken, C. Adamo, J. Jaramillo, R. Gomperts, R. E. Stratmann, O. Yazyev, A. J. Austin, R. Cammi, C. Pomelli, J. W. Ochterski, P. Y. Ayala, K. Morokuma, G. A. Voth, P. Salvador, J. J. Dannenberg, V. G. Zakrzewski, S. Dapprich, A. D. Daniels, M. C. Strain, O. Farkas, D. K. Malick, A. D. Rabuck, K. Raghavachari, J. B. Foresman, J. V. Ortiz, Q. Cui, A. G. Baboul, S. Clifford, J. Cioslowski, B. B. Stefanov, G. Liu, A. Liashenko, P. Piskorz, I. Komaromi, R. L. Martin, D. J. Fox, T. Keith, M. A. Al-Laham, C. Y. Peng, A. Nanayakkara, M. Challacombe, P. M. W. Gill, B. Johnson, W. Chen, M. W. Wong, C. Gonzalez and J. A. Pople, Gaussian 03, Gaussian, Inc., Wallingford, CT, (2004).
[2] A. J. Misquitta and A. J. Stone, CamCASP: a program for studying intermolecular interactions and for the calculation of molecular properties in distributed form, (2007), wwwstone.ch.cam.ac.uk/programs.html\#CamCASP.
[3] A. J. Stone, Chem. Phys. Lett. 83 (1981) 233.
[4] A. J. Stone and M. Alderton, Molec. Phys. 56 (1985) 1047.
[5] A. J. Stone, The Theory of Intermolecular Forces, International Series of Monographs in Chemistry, (Clarendon Press, Oxford, 1996).
[6] S. F. Boys, Proc. Roy. Soc. A 200 (1950) 542.
[7] A. J. Stone, J. Chem. Theory Comput. 1 (2005) 1128.
[8] V. I. Lebedev and D. N. Laikov, Doklady Mathematics 59 (1999) 477.
[9] A. D. Becke, J. Chem. Phys. 88 (1988) 2547.
[10] C. W. Murray, N. C. Handy and G. J. Laming, Molec. Phys. 78 (1993) 997.
[11] J. C. Slater, J. Chem. Phys. 41 (1964) 3199.
[12] A. J. Stone, A. Dullweber, O. Engkvist, E. Fraschini, M. P. Hodges, A. W. Meredith, D. R. Nutt, P. L. A. Popelier and D. J. Wales, ORIENT: a program for studying interactions between molecules, version 4.6, University of Cambridge, (2006), http://wwwstone.ch.cam.ac.uk/programs.html\#Orient.
[13] P. J. Winn, G. G. Ferenczy and C. A. Reynolds, J. Phys. Chem. A 101 (1997) 5437.
[14] G. G. Ferenczy, P. J. Winn and C. A. Reynolds, J. Phys. Chem. A 101 (1997) 5446.

